Recognizing Intentions By Fuzzy Abductive Reasoning

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ABSTRACT
This paper proposes a method to infer intentions of users of computer systems that have interactive man-machine interface. The main feature of the approach is the employment of fuzzy abduction -- inference procedure to find an appropriate explanation of given fuzzy events. The paper describes a user utterance model expressed by a goal hierarchy whose goals are different levels of intentions, and introduces fuzzy abduction as a mechanism to infer intentions. Then, it shows an example to recognize user's intentions from his utterance.

INTRODUCTION
A major problem in the area of user-computer interaction is the lack of natural communication. Person-machine dialogues do not have the "cooperative" feel that person-person dialogues do. The essential difference between these two types of communication is in how responses are generated. Computers respond just to the user's literal question, while human beings respond both to the question and to the speaker's intentions.

Plan-based theory of speech acts\(^1\) is a major approach that tries to model speech acts. It treats user's intention as a plan expressed as a sequence of actions and shows how plans can link speech acts with nonlinguistic behavior. Allen & Perrault\(^2\) and Litman & Allen\(^3\) extended the model to realize several kinds of helpful responses done by human beings.

In the model, intentions (plans) are inferred by domain-independent rules and knowledge about actions such as preconditions, effects, etc. Though the model itself is well devised, the inference used is very simple. It uses the rules in both forward and backward directions by simple pattern matching. This may work in inference of low-level intentions but it might not in general. It is risky to use rules such as "if A wants to execute an action, then A wants the effects of that action"\(^2\) in the backward manner.

Recognition of intentions is the procedure to derive possible hypotheses (motivations) from observed events. Such procedure of inference is inherently abduction\(^4-7\).

Furthermore, we introduce the fuzzy set theory to abduction. That is because meanings of utterances or direct intentions, which should be obtained in preprocessing of intention inference, are frequently vague. Fuzzy sets are convenient to express such vague meanings.

USER UTTERANCE MODEL
We assume that every user's utterance has a primary goal (primary intention) which is organized as a goal hierarchy. The goals in the hierarchy are expressed as actions which the speaker wants to do. When an utterance is observed, a terminal of the hierarchy is its direct intention. The goals between the primary and direct intentions are indirect intentions.

The goal hierarchy is built by interpreting intention inference rules with truth values in \([0, 1]\). The rules used in the lower level could be domain-independent like those in the plan-based approach\(^2\). Rules in higher level, however, are almost domain-dependent, because decompositions of higher level of intentions depend highly on domain knowledge.

Then, intention recognition here is interpreted as one or all of the following procedures:
1) to infer the primary intention
2) to infer indirect intentions
3) to infer direct intentions

Naturally from the structure of intentions, 1) needs 2), and 2) needs 3).

The final goal of intention recognition is of course 1). For simplicity and reality, however, we remove it from our definition, that is, we do not build the whole hierarchy, but a lower part of the hierarchy corresponding to the user's utterances. Furthermore, we regard 3) as the task of natural language understanding. So, by intention recognition we mean to find appropriate indirect intentions from given direct ones. Which level of indirect intentions are inferred is dependent on the purpose of intention recognition.

FUZZY ABDUCTION\(^7\)
In modus ponens of multi-valued logics, the truth value of the conclusion is generally obtained as an interval. If you follow Lukasiewicz Infinite valued Logic(\(\text{ALEPH-1}\)) and truth values are given as \(|A \rightarrow B| = \tau = 1.0\) and \(|A| = a < 1.0\), then \(|B| = \beta\) takes a value in \([\alpha, 1.0]\). The same thing happens even if you choose another MVL such as Godel, Goguen, Rescher, etc. So, we start with introduction of a peculiar inference
named Causal Inference (CI), which gives a single truth to the conclusion.

**Definition 1: Causal Inference**

The procedure to obtain \( b \) from \( r \) and \( a \) in the situation where the following conditions hold is called Causal Inference (CI).

\[
\begin{align*}
  b > 0.0 & \quad \text{iff } a > 0 \text{ and } r > 0, \\
  b = 0.0 & \quad \text{otherwise}.
\end{align*}
\]

This inference is observed, when \( B \) is a proposition which cannot be true without any cause and there is no other implication that concludes \( B \) except \( A \rightarrow B \).

The truth value obtained by CI is called CI-truth value (CTV).

**Assumption 1**

CTV of \( B \), \( b^* \) calculated from \( a \) and \( r \) should satisfy the followings:

\[
\begin{align*}
  b^* &= a \quad \text{if } r = 1, \\
  b^* &= a \text{ and } b^* < r \quad \text{if } b \in \emptyset, 0 < r < 1, a \neq 0, \\
  b^* &= 0 \quad \text{if } b \in \emptyset, r x a = 0, \\
  b^* &= \emptyset \quad \text{if } b = \emptyset.
\end{align*}
\]

The basic idea of the assumption is that \( b^* \) should not be greater than \( r \) and \( a \) if there is no other implication that concludes \( B \).

From now, let us choose \( L_{ALEPH-1} \) as the base logic for CI. In \( L_{ALEPH-1} \), implication is defined as:

\[
\begin{align*}
  r &= 1 = a + b \land 1. \quad (3)
\end{align*}
\]

Modus ponens is derived from Eq. (3) as:

\[
\begin{align*}
  [a, 1] & \quad \text{if } r = 1, \\
  b &= a + r - 1 \quad \text{if } 1 \leq a + r \leq 2, r = 1, \\
  & \quad \emptyset \quad \text{otherwise}.
\end{align*}
\]

Then, CTV by CI on \( L_{ALEPH-1} \) is obtained as a single value:

\[
\begin{align*}
  b^* &= a + r - 1 \quad \text{if } 1 \leq a + r, \\
  & \quad \emptyset \quad \text{otherwise}.
\end{align*}
\]

**Definition 2: Inverted CI**

The procedure to obtain \( a \) from \( r \) and \( b \) in the situation where the conditions (1) hold is called Inverted Causal Inference (ICI).

The truth value obtained by ICI is called Desirable Truth Value (DTV). In order to obtain DTV, we employ modus tollens of \( L_{ALEPH-1} \):

\[
\begin{align*}
  [0, b] & \quad \text{if } r = 1, \\
  a &= b - r + 1 \quad \text{if } 0 < b < 1, r = 1, \\
  & \quad \emptyset \quad \text{otherwise}.
\end{align*}
\]

and introduce the next assumption.

**Assumption 2**

DTV of \( A \), \( a^* \) calculated from \( b \) and \( r \) should satisfy the followings:

\[
\begin{align*}
  a^* &= b \quad \text{if } r = 1, \\
  a^* &= b \quad \text{if } a \neq \emptyset, 0 < r < 1, b = 0, \\
  a^* &= 0 \quad \text{if } a = \emptyset, r x b = 0,
\end{align*}
\]

From Eq. (6) and assumption 2, we obtain DTV as:

\[
\begin{align*}
  a^* &= b - r + 1 \quad \text{if } b \leq r, \\
  & \quad \emptyset \quad \text{otherwise}.
\end{align*}
\]

Notice that CTV is the smallest truth value that is consistent with modus ponens, and that DTV is the largest truth value that is consistent with modus tollens.

**Definition 3: Derivable**

When \( a, b \) and \( r \) are given and \( b^* = b \neq \emptyset \), then \( B \) is derivable from \( A \) and \( A \rightarrow B \).

**Definition 4: Derivable fuzzy set**

Let \( R, P \) and \( Q \) denote a set of implications, their antecedents and their consequents, respectively. We express implications as:

\[
\begin{align*}
  R_{ij} : P_{ij} \rightarrow Q_i, i = 1, N_q, j = 1, N_{q_i}, \\
  R_{ij} \in R, P_{ij} \in P, Q_i \in Q.
\end{align*}
\]

R is given in disjunctive form and \( P_{ij} \neq P_{ij}^2 \) for \( j \neq j^2 \). Then, the fuzzy set \( Q \) on \( Q \) is derivable from \( R \) and fuzzy set \( P \) on \( P \) if the next equation is satisfied:

\[
\begin{align*}
  Q(Q_i) &= \max (q_{ij}^*) \\
  &= \max (p_{ij} + r_{ij} - 1),
\end{align*}
\]

where \( Q(Q_i) = q_{ij} \) and \( p_{ij} \) is the memberships of \( Q_i \) in \( Q \) and \( P_{ij} \) in \( P \), respectively. If Eq. (5) is replaced by \( b^* = a \land r \), then Eq. (10) becomes the Max-Min composition. Clearly from Eq. (8), fuzzy set \( Q \) is not derivable from any fuzzy set on \( P \) unless the next condition is satisfied:

\[
\exists (Q_i) \text{ s.t. } \forall (R_{ij}, P_{ij}, Q_i), |R_{ij}| > 2( Q_i )
\]

Then, the following theorem holds. (Proofs are omitted due to lack of space.)

**Theorem 1**

Given a set of implications \( R = \{ R_{ij} \} \) and a fuzzy set \( Q \) on \( Q = \{ Q_i \} \), then

(1) If there exists any fuzzy set \( P \) on \( P = \{ P_{ij} \} \) which can derive \( Q \), \( P^* \) given by Eq. (12) is the largest fuzzy set among them.

\[
\begin{align*}
  P^* &= \emptyset, \\
  p_{ij}^* &= \max (p_{ij}^* \land P_{ij}^*), \quad (12)
\end{align*}
\]

where \( p_{ij}^* \) is DTV calculated from \( r_{ij} \) and \( q_i = Q(Q_i) \), and \( \land \) is an operator defined below.

\[
\begin{align*}
  a / A \land b / B &= a / A + b / B \quad \text{if } A \land B. \quad (13)
\end{align*}
\]

(2) \( P^* \) given by Eq. (14) is a minimal fuzzy set on \( P \) which derives \( Q \), if \( P^* \) is a subset of \( P^* \).

\[
\begin{align*}
  P^* &= \sum (j) \left( \Delta_l(\Delta_j^l / P_{ij}^l) \right), \quad (14)
\end{align*}
\]

where \( \Delta \) is an operator to pick a term whose \( p_{ij}^* \) \( \neq \emptyset \) among those with different "j".

(3) When \( P^* \) and \( P^* \) are the largest and a minimal fuzzy sets which derives \( Q \), any fuzzy set \( P \) satisfying \( P^* \leq P \leq P^* \) can derive \( Q \).

---

1. \( b = \emptyset \) means that there is no \( b \) consistent with given \( a \) and \( r \).
2. You may choose other MVL such as Godel, Goguen and Rescher.
3. CTV is also a single value in other MVLs.
4. When \( A \) and \( B \) are fuzzy sets on \( X \), \( A \leq B \) iff \( A(x) \leq B(x) \) for all \( x \in X \), and \( A = B \) iff \( A(x) = B(x) \) for all \( x \in X \).
Now, we are prepared to define fuzzy explanation and fuzzy abduction.

**Definition 5 : Fuzzy explanation**

Fuzzy set $P$ is a fuzzy explanation of fuzzy set $Q$ by $R$ and $C$, if

1) $Q$ is derivable from $R$ and $P$
2) $P$ is consistent with $C$,

where $C_{m}$ is a set of constraints $C_{m}$ given below.

$$C_{m} : | \bigwedge_{n} P_{n}^{m} | \leq T_{m}, \quad (15)$$

$m=1,N_{c}, n=1,N_{cm}, P_{n}^{m} \subseteq P,T_{m} \in [0,1].$

**Definition 6 : Fuzzy abduction**

Fuzzy abduction is a procedure to obtain a fuzzy explanation.

Notice that existence of the largest fuzzy explanation is NOT guaranteed even if $P^{*}$ exists because of the constraints $C$. It is, however, easy to get maximal fuzzy explanations from $P^{*}$ and $C$ since the constraints are limited to maximum truth values of conjunctions of antecedent propositions. Then, it is guaranteed that $P$ which can derive $Q$ and is included in a maximal fuzzy explanation is a fuzzy explanation, and that only such $P$s are fuzzy explanations.

We assume that the best fuzzy explanation is that which has the minimum cardinality among the minimal ones based on the idea of minimal covering.

**CASES OF COMPOSITE ANTECEDENTS**

In this section, we discuss cases where antecedents of implications are composite propositions.

Suppose $R_{i} \in R$, $S_{i} = \bigwedge P_{ij} \in S$ and $S = \sum_{i}(s_{i}/S_{i})$ be an implication, composite antecedent proposition of $R_{i}$ and a minimal fuzzy explanation on $S$ of $Q$ by $R$ and $C$, respectively. Then, fuzzy explanation $P$ on $P = \{P_{ij}\}$ of $Q$ by $R$ and $C$ should satisfy Eq. (16) below:

$$D_{i} : | \bigwedge P_{ij} | \geq s_{i}. \quad (16)$$

The minimum fuzzy set $P^{*}$ on $P$ that satisfies Eq. (16) is obtained as:

$$P^{*} = \sum_{i}(s_{i}/P_{ij}). \quad (17)$$

Since $S^{*}$ calculated from $P^{*}$ and $S_{i} = \bigwedge P_{ij}$ is equal to or greater than the minimal fuzzy explanation $S$, $P^{*}$ is a fuzzy explanation on $P$ if $S^{*}$ is included in a maximal fuzzy explanation. Furthermore, $P^{*}$ is a minimal fuzzy explanation if $S^{*}=S$, or $P^{*}$ does not include $P^{*}$ which is obtained from another minimal fuzzy explanation $S^{*}$.

**EXAMPLE**

$R=[R_{i}]_{i=1,7}$, $C=[C_{1}]$ and a fact $Q$ are given as follows:

- $R_{1}$: if $P_{1} \land P_{2}$ then $Q_{1}$ ($T_{1}=0.8$)
- $R_{2}$: if $P_{3}$ then $Q_{1}$ ($T_{2}=0.5$)
- $R_{3}$: if $P_{4}$ then $Q_{1}$ ($T_{3}=0.9$)
- $R_{4}$: if $P_{2} \land P_{3}$ then $Q_{2}$ ($T_{4}=0.6$)
- $R_{5}$: if $P_{5}$ then $Q_{2}$ ($T_{5}=0.8$)
- $R_{6}$: if $P_{3} \land P_{4}$ then $Q_{3}$ ($T_{6}=0.5$)
- $R_{7}$: if $P_{1}$ then $Q_{3}$ ($T_{7}=0.7$)
- $C_{1} : | P_{4} \land P_{5} | \leq 0.6$
- $Q=0.6/Q_{1}+0.5/Q_{2}+0.6/Q_{3}$

Let us suppose antecedents of $R_{i}$ be $S_{i}$ Then, from Eq. (12) and (14) we get the largest fuzzy set $S^{*}$ and minimal fuzzy sets $S_{i}^{*}$ on $S$ which derive $Q$ as follows.

- $S_{1}^{*}=.8/S_{1}+.7/S_{3}+.9/S_{4}+.7/S_{5}+.9/S_{7}$
- $S_{1}^{*}=.8/S_{1}+.9/S_{4}+0.7/S_{7}$
- $S_{2}^{*}=.8/S_{1}+.7/S_{3}+.9/S_{4}+.6/S_{5}+.9/S_{7}$
- $S_{4}^{*}=.7/S_{3}+.7/S_{5}+.9/S_{7}$

Constraint $C_{i}$ can be replaced by

- $C_{1}^{*} : | S_{3} \land S_{5} | \leq 0.6$
- $C_{2}^{*} : | S_{5} \land S_{6} | \leq 0.6$

So, maximal fuzzy explanations on $S$ are

- $S_{1}=0.8/S_{1}+.6/S_{3}+.9/S_{4}+.7/S_{5}+.9/S_{7}$
- $S_{2}=0.8/S_{1}+.7/S_{3}+.9/S_{4}+.6/S_{5}+.9/S_{7}$
- $S_{4}=0.7/S_{3}+.7/S_{5}+.9/S_{7}$

Constraint $C_{1}$ is not a fuzzy explanation, because it is included neither in $S_{1}$ nor $S_{2}$. Then, by $S_{1}^{*}$, $S_{2}^{*}$, $S_{3}^{*}$ and Eq. (17), we get the following possible minimal explanations on $P$

- $P_{1}^{*}=0.9/P_{1}+0.9/P_{2}+0.9/P_{3}$
- $P_{2}^{*}=0.9/P_{1}+0.9/P_{2}+0.9/P_{3}$
- $P_{3}^{*}=0.9/P_{1}+0.9/P_{2}+0.9/P_{3}$

However, since $S_{1}^{*}$ and $S_{3}^{*}$ calculated from $P_{1}^{*}$ and $P_{3}^{*}$ are not included in $S_{1}^{*}$ and $S_{2}^{*}$. Only $P_{2}^{*}$ is a minimal fuzzy explanation of $Q$.

**OUTLINE OF INTENTION RECOGNITION**

Let us consider a real estate agency who tries to infer a customer’s preferences (higher-level indirect intentions) from his questions about recommended apartments. At first, we infer the customer’s requirements (lower-level indirect intentions) from his questions (direct intentions) using domain-independent rules such as:

- IF want(subj:x, obj:(y,c))(is(subj:a(y),comp:p))
  THEN know(subj:x, if:is(subj:a(z,c),comp:p)),

with truth=T1.

- IF want(subj:x, obj:(y,c))
  THEN know(subj:x, that:is(subj:a(y),comp:p)),

with truth=T2.

The first rule means that a person x who wants such an instance y of class c that its attribute a is p might ask whether the attribute a is an instance of class c is p. The second means that a person x who wants y might ask about attributes of y. With the first one, the agency can estimate that a customer wants an apartment in a residential area from his question to ask if a recommended apartment exists in a residential area. The inference is done in the backward manner by simple pattern matching or fuzzy abduction described above.

After all requirements of the customer are obtained, the customer’s preferences are then inferred by fuzzy abduction using the following rules. Those rules
have preferences in their antecedents and requirements in consequents.

R1: Good-looking -> New (1.0)
R2: Good-looking -> Tiled-wall (0.9)
R3: Good-looking -> Remote-lock-system (0.8)
R4: Safety -> Remote-lock-system (1.0)
R5: Safety -> Caretaker (0.9)
R6: Safety -> Residential-area (1.0)
R7: Comfortable -> Caretaker (0.8)
R8: Comfortable -> Basement-garage (0.8)
R9: Silent -> Residential-area (1.0)
R10: Silent -> Sound-proof (0.8)

Here, R1 means that a customer who prefers a good-looking apartment requires a new one. Then, suppose that requirements of the customer have been obtained as follows.

Q1: New (0.9)
Q2: Tiled-wall (0.8)
Q3: Remote-lock-system (0.7)
Q4: Caretaker (0.6)
Q5: Residential-area (0.7)
Q6: Basement-garage (0.6)
Q7: Sound-proof (0.5)

Now, the real estate agency can infer the customer's preference which is a fuzzy subset on {Good-looking, Safety, Comfortable, Silent}. By applying fuzzy abduction to this problem and assuming that the best fuzzy explanation is the one with the minimum cardinality, he can get the following customer's preference.

\[ P = 0.9/\text{Good-looking} + 0.8/\text{Comfortable} + 0.7/\text{Silence} \]

CONCLUSION

A new idea to recognize user's intentions by fuzzy abduction was shown. The plan-based theory of speech act is powerful to infer intentions in simple tasks such as conversations at an information desk. Its inference mechanism is, however, too simple to apply it to more complex applications such as decision support systems, where users have several and complex intentions in general. So, we proposed a fuzzy abduction to infer intentions in such complex systems.

In the introduction of the fuzzy abduction, semantics of implication in multi-valued logics was examined, inference in a certain situation and fuzzy explanation were defined, and a way to infer fuzzy explanation was shown.

Finally, the method was applied to a few simple examples and the effectiveness was demonstrated.

REFERENCES

8) Y. Tsukamoto: Fuzzy Logic Based on Lukasiewicz Logic and Its Applications to Diagnosis and Control, Doctoral Thesis, Tokyo Institute Technology (1979)

APPENDIXES

The following theorems for derivable fuzzy sets hold, though proofs are omitted due to lack of space.

**Theorem 2**
Given \( R = \{ r_{ij} \} \) and \( Q \) on \( Q = \{ q_{ij} \} \), \( P \) on \( P = \{ p_{ij} \} \) that derives \( Q \) exists iff
1) For any \( i \) such that \( q_{ij} = 0 \),
   \[ \exists j_0 \text{ s.t. } q_{ij} = p_{ij} + r_{ij} - 1, \]
2) For any \( i \) such that \( q_{ij} = 0 \),
   \[ \forall j : 1 > p_{ij} + r_{ij}, \]
   where \( p_{ij} \) is the membership of \( P \) in \( P \) given by Eq. (12).

**Theorem 3**
There exists only a minimal fuzzy set \( P \) on \( P \) which derives \( Q \), iff there exists only \( j \) (\( = j_i \)) for any \( i \) that satisfies the following 1) and 2).
1) \( r_{ij} \geq q_{ij} \)
2) For any \( i' = i \) and \( j' = j \) that satisfies \( r_{ij'} - q_{ij'} \),
   \[ P_{ij} = P_{ij'}, \text{ or } P_{ij} = P_{ij'} \text{ and } q_{ij} - r_{ij} + 1 = q_{ij'} - r_{ij'} + 1. \]

**Theorem 4**
If there exists only a minimal fuzzy set \( P \) which derives \( Q \), it is the unique fuzzy set on \( P \) which derives \( Q \).