ABSTRACT

Causality, uncertainty and intensity of occurrence are three important factors we must take into consideration, when we analyze complex problems in the world. Causality is described as "an essential element of human understanding of all phenomena in the world", and has been used as important knowledge to analyze many real world problems. Uncertainty and intensity of occurrence, on the other hand, are regarded as origins of complexity in analyzing problems. This paper addresses a general causal reasoning with uncertainty and intensity. The paper first defines Conditional Causal Possibility that expresses exact uncertainty of causality, and proposes Causality Analysis Reasoning based on Possibility theory. CAR is a way of reasoning with a hierarchical causal network, and calculates possibilities of arbitrarily chosen unknown events when some events in the network are observed. Then, it additionally introduces intensity of occurrence to the possibilistic CAR.

1. INTRODUCTION

Causality, uncertainty and intensity of occurrence are three important factors we must take into consideration, when we analyze complex problems in the world. Causality is described as "an essential element of human understanding of all phenomena in the world" [1], and has been used as important knowledge to analyze many real world problems. Uncertainty and intensity of occurrence, on the other hand, are regarded as origins of complexity in analyzing problems.

In general, reasoning with causal knowledge is divided into three types: 1) reasoning effects from observed causes, 2) reasoning causes from observed effects, and 3) reasoning any unknown events from observed causes and effects. The first one is conducted by deduction, and sometimes referred to as predictive reasoning. The second is done by abduction, and referred to as diagnostic reasoning. The last is the most general one with both predictive and diagnostic aspects. The paper calls it Causality Analysis Reasoning (CAR), and addresses it.

As for uncertainty of causality, there is a long and rich history of studies using Probability theory. Peng and Reggia proposed Conditional Causal Probability to express uncertainties of causalities, and applied them to a diagnostic problem [2]. Bayesian Networks deal with more general CAR in uncertain environments using a Directed Acyclic Graph [3]. In addition, there are some new studies employing Possibility theory [10,11], which is another uncertain measure suitable for ordering events according to their uncertainties [4,5].

Intensity of occurrence, which is sometimes expressed in fuzziness, has been also studied in the context of causal reasoning. Fuzzy Cognitive Maps conducts analysis through simulations using fuzzy causalities [6], and Fuzzy Abductive Reasoning finds fuzzy causes from fuzzily observed effects using fuzzy causal knowledge [7,8]. Intensity reasoning that uses the causal knowledge both in predictive and diagnostic manner has also been proposed [9].

There are many studies addressing one of the three types of causal reasoning either with uncertainty or intensity as mentioned above. However, there are few that cover both of them. This paper proposes a new way to conduct CAR both with uncertainty and intensity. In the proposed reasoning, causalities given as knowledge have distributions assigning uncertainty to each intensity degree of occurrence.

After the introduction, the paper defines Conditional Causal Possibility similar to Conditional Causal Probability, and proposes a CAR based on Possibility theory. Then, it introduces intensity to the possibilistic CAR. A numerical example is shown at the end of the paper.

2. CONDITIONAL CAUSAL POSSIBILITY

Let X be a set of variables $x_i$ ($i=1, \ldots, n$) and Y be a set of $y_j$ ($j=1, \ldots, m$). $x_i$ and $y_j$ express a state of objects $i$ and $j$, and their generic values are denoted by $u_i$ and $v_j$, which are elements of finite sets $U_i$ and $V_j$, respectively. $y_j$ is possibilistically (or probabilistically) dependent on $x_i$.

[Definition 2.1]
Generic values $u_i, v_j$ are called cause and result events,
If and only if the next equation is satisfied,

\[ v_j : u_j \iff (v_j : u_j) \land u_j \iff (v_j : u_j) \land v_j \]

(1)

where \( \land, \iff \) are logical product and equivalence.

Possibility Theory is introduced here. Let marginal possibility distributions on \( U_i \) and \( V_j \) be \( \pi(x) \) and \( \pi(y) \) respectively, and possibility distribution of \( v_j : u_j \) on \( U_i \land V_j \) be \( \pi(v_j : x) \). Possibilities of their generic values are denoted by \( \pi(u), \pi(v) \) and \( \pi(v_j : u_j) \), respectively.

**Definition 2.2**
Conditional possibility \( \pi(v_j : u_j | u_j) \) of \( v_j : u_j \) conditioned by \( u_j \) is called **Conditional Causal Possibility**.

**Definition 2.3**
If and only if the next condition holds, \( x_i \) and \( y_j \) have causality.

\[ \exists (u_i, v_j) \in (U_i \land V_j) \land \pi(v_j : u_j | u_j) > 0.0. \]

(2)

Now, the next equation holds in general [11].

\[ \pi(v_j \land u_j) = \pi(u_j) \land \pi(v_j | u_j) \]  

(3)

In the paper, \( \land(v) \) means \( \min(\max) \) if it is used for possibilities, while it expresses logical product (sum) for events as noted before.

\( \pi(v_j \land u_j) \) is equivalent to \( \pi(v_j, u_j) \).

From the above, the following is derived.

\[ \pi(v_j : u_j) = \pi(v_j : u_j | u_j) \land u_j = \pi(u_j) \land \pi(v_j | u_j). \]

(4)

**Definition 2.4**
Let \( A_y \) be a conjunction of cause events and causation events. If \( A_y \) does not include cause events of \( x_i \), and causation events by \( x_i \) and \( y_j \), it is called **context** of \( v_j : u_j \).

**Definition 2.5**
Let \( A_y \) be an arbitrary context of \( v_j : u_j \). If the next equation is satisfied, \( v_j : u_j \) is **possibilistically causal** independent (PCI).

\[ \pi(v_j : u_j | u_j \land A_y) = \pi(v_j : u_j | u_j) \]

(5)

**Definition 2.6**
If and only if the equation below is satisfied, \( x_i \) is **possibilistically independent** of \( x_j \)[11].

\[ \pi(x_j | x_i) = \pi(x_j) \]

(6)

**Definition 2.7**
If and only if the next equation is satisfied, \( x_i \) and \( x_j \) are **non-interactive**[10].

\[ \pi(x_j \land x_j) = \pi(x_j) \land \pi(x_j) \]

(7)

If \( x_i \) and \( x_j \) are possibilistically independent of each other, they are non-interactive[10].

**Definition 2.8**
If and only if the next equation holds, \( v_j \) satisfies the **Mandatory Causation Assumption** (MCA).

\[ v_j \iff \bigvee_{u_j \in U_i \land V_j} (v_j : u_j). \]

(8)

**Assumption 2.1**
All \( x_i \)s are possibilistically independent of each other.

All causation events are PCI.

All result events satisfy the MCA.

From these assumptions, we get the following:[5]

\[ \pi(v_j : u_j | u_j) = \pi(v_j : u_j), \quad i \neq k. \]

(9)

\[ \pi(v_j : u_j | v_k : u_k) = \pi(v_j : u_j), \quad i \neq k. \]

(10)

\[ \pi(v_j : u_j \land v_k : u_k | B) = \pi(v_j : u_j | B) \land \pi(v_k : u_k | B). \]

(11)

\[ \pi(v_j) = \bigvee_{u_j \in U_i \land V_j} \pi(v_j : u_j) \land \pi(u_j) \]

(12)

\[ \pi(v_j \land u_j \land \cdots \land u_p) = \bigvee_{u_j \in U_i \land V_j} \pi(v_j : u_j) \land \pi(u_j) \land \pi(u_p). \]

(13)

\[ \pi(v_j | u_1 \land \cdots \land u_p) = \bigwedge_{j=1} \pi(v_j | u_1 \land \cdots \land u_p). \]

(14)

In the above, \( B \) is a conjunction of cause events, and \( p \) is.

**3. POSSIBILISTIC CAUSALITY ANALYSIS REASONING**

**3.1 Problem definition**
Possibilistic Causality Analysis Reasoning is defined as follows.

**Definition 3.1**
Suppose \( x_i = x_i^p \) (\( i \leq p \leq n \)) and \( y_j = y_j^q \) (\( j \leq q \leq m \)). Values of other \( x_i \)s and \( y_j \)s, are unknown. **Possibilistic Causality Analysis Reasoning** is one to obtain possibility distributions of \( x_i \) (\( i=p+1, \ldots, n \)) and \( y_j \) (\( j=q+1, \ldots, m \)) conditioned by the known events; \( \pi(x_i | u_i \land \cdots \land u_i \land y_j \land \cdots \land y_j) \) and \( \pi(y_j | u_j \land \cdots \land u_j \land y_j \land \cdots \land y_j) \), where \( \pi(x_i) \) and \( \pi(y_j | x_i) \) satisfying the next assumptions are given as a priori knowledge.

**Assumption 3.1**

1. \( \pi(x) \) is normal and convex.
2. Every \( x_i (y_j) \) has causality at least with a \( y_j (x_i) \).
3. If \( x_i \) and \( y_j \) have causality, possibility distribution \( \pi(y_j : u_j | u_j) \) on \( V_j \) is normal and convex for any \( u_j \).

**3.2 Conditional Possibilities of \( v_i \)**
From eq. (3), the next equation holds in general.

\[ \pi(v_j | u_i \land \cdots \land u_i \land y_j \land \cdots \land y_j) \land \pi(u_i \land \cdots \land u_i \land y_j \land \cdots \land y_j) \]

\[ = \pi(v_j \land y_j | u_i \land \cdots \land u_i \land y_j \land \cdots \land y_j) \]

(15)

Let \( E \) and \( F(y) \) be given as follows:
Thus, we get the next equation using the Least Specific Solution.

\[ E = \bigwedge_{j=1,q} \pi(v_j^R \land \ldots \land v_j^D \mid u_j^R \land \ldots \land u_j^D) \land \bigwedge_{i=1,p} \pi(u_i^R \land \ldots \land u_i^D) \]

\[ = \bigwedge_{j=1,q} \pi(v_j^R \mid u_j^R \land \ldots \land u_j^D) \land \bigwedge_{i=1,p} \pi(u_i^R) \tag{16} \]

\[ F(y_j) = \pi(v_j^R \land \ldots \land v_j^D \mid y_j \mid u_j^R \land \ldots \land u_j^D) \land \pi(u_j^R \land \ldots \land u_j^D) \]

\[ = \bigwedge_{j=1,q} \pi(v_j^R \mid u_j^R \land \ldots \land u_j^D) \land \pi(y_j \mid u_j^R \land \ldots \land u_j^D) \land \bigwedge_{i=1,p} \pi(u_i^R) \tag{17} \]

Though we can derive \( \pi(y_j \mid u_j^R \land \ldots \land u_j^D \land v_j^R \land \ldots \land v_j^D) \) from eq. (15) as an interval value, it is appropriate to choose the upper limit of the interval according to an idea of the Least Specific Solution [12]. Thus, we get the equation below.

\[ \pi(y_j \mid u_j^R \land \ldots \land u_j^D \land v_j^R \land \ldots \land v_j^D) = \begin{cases} F(y_j), & \text{if } E > F(y_j), \\ 1, & \text{if } E = F(y_j). \end{cases} \tag{18} \]

The largest value of the above equation is always 1.0, and the set \( T \) of values of \( y_j \) giving 1.0 is given as follows [5]:

\[ T = \bigcup_{i=1,n} T_i \tag{19} \]

\[ T_i = \begin{cases} \{ v_j \mid \pi(v_j \mid u_j^R \mid u_j^D) \geq E \}, & \text{if } i \in \{1, \ldots, p\}, \\ \{ v_j \mid \pi(v_j \mid u_j \mid u_j) \geq E \}, & \text{if } i \in \{p+1, \ldots, n\}. \end{cases} \tag{20} \]

### 3.3 Conditional Possibilities of \( u_i \)

The next equation holds in general.

\[ \pi(x_j \mid u_j^R \land \ldots \land u_j^D \land v_j^R \land \ldots \land v_j^D) \land \pi(v_j^R \land \ldots \land v_j^D \mid u_j^R \land \ldots \land u_j^D \land x_j) \]

\[ = \pi(v_j^R \land \ldots \land v_j^D \mid u_j^R \land \ldots \land u_j^D \land x_j) \land \pi(u_i^R \land \ldots \land u_i^D \land x_j) \]

\[ \land \pi(u_i^R \land \ldots \land u_i^D \land x_j) \land \pi(x_j) \tag{21} \]

Let \( G(x_j) \) be

\[ G(x_j) = \pi(v_j^R \land \ldots \land v_j^D \mid u_j^R \land \ldots \land u_j^D \land x_j) \land \pi(u_i^R \land \ldots \land u_i^D \land x_j) \land \pi(x_j) \]

\[ = \bigwedge_{j=1,q} \pi(v_j^R \mid u_j^R \land \ldots \land u_j^D \land x_j) \land \bigwedge_{i=1,p} \pi(u_i^R) \land \pi(x_j). \tag{22} \]

Thus, we get the next equation using the Least Specific Solution.

\[ \pi(x_j \mid u_j^R \land \ldots \land u_j^D \land v_j^R \land \ldots \land v_j^D) = \begin{cases} G(x_j), & \text{if } E > G(x_j), \\ 1, & \text{if } E = G(x_j). \end{cases} \]

In this case, it is not guaranteed that the largest possibility is 1.0. The set \( S \) of values of \( x_j \) giving the largest value is given as follows [5]:

\[ S = \bigcap_{j=0,q} S_j \]

\[ S_j = \begin{cases} \{ u_x \mid \pi(u_x) \geq G_{\max} \}, & \text{if } j = 0, \\ \{ u_x \mid \pi(v_j^R : u_j \mid u_j) \geq G_{\max} \}, & \text{if } j \in \{1 \leq j \leq q, H_j \geq G_{\max}\}. \end{cases} \tag{24} \]

\[ H_j = \bigvee_{i=1,p} \pi(v_j^R : u_j \mid u_j) \]

\[ \bigvee_{u_j \in U_j} \left( \pi(v_j^R : u_j \mid u_j) \land \pi(u_j) \right). \tag{26} \]

\[ G_{\max} = \bigvee_{u_j \in U_j} G(u_j). \tag{27} \]

### 4. CAR WITH UNCERTAINTY AND INTENSITY

#### 4.1 Introduction of Intensity

From now, we assume that \( U_i \) and \( V_j \) are totally ordered sets.

[Definition 4.1]

When \( U_i \) and \( V_j \) are totally ordered sets \((U_i, \leq), (V_j, \leq)\), \( u_i \) and \( v_j \) express intensities of objects \( i \) and \( j \), respectively.

[Definition 4.2]

When subset \( A \) of \((U, \leq)\) satisfies the next equation, \( A \) is an interval, and expressed by \([a_{\min}, a_{\max}]\), where \( a_{\min} \) is the minimum value and \( a_{\max} \) is the maximum.

\[ \forall u, w \in A \subseteq U \Rightarrow \forall v \in [v \leq w \leq v], v \in A. \tag{28} \]

[Definition 4.3]

When fuzzy subset \( A \) on \((U, \leq)\) satisfies the next equation, \( A \) is convex.

\[ \forall u, v, w \in U, u < v < w \Rightarrow A(v) \leq A(u) \land A(w), \tag{29} \]

where \( A(u) \) is a membership value of \( u \) in \( A \).

[Definition 4.4]

Suppose that fuzzy subsets \( A, B \) on \((U, \leq)\) are given as follows:

\[ A = \sum_u A(u) \bigg/ u, \ u \in U, \tag{30} \]
\[ B = \sum_{v} B(v) / v, \quad v \in U. \]  

(31)

Then, binary relation \( A \leq B \) and \( A \leq B \) are defined as shown below.

\[ A \leq B = \sum_{u,v} (A(u) \land B(v)) / (u \land v). \]  

(32)

\[ A \leq B = \sum_{u,v} (A(u) \land B(v)) / (u \land v). \]  

(33)

[Definition 4.5]

When fuzzy subsets \( A \) and \( B \) on \((U, \preceq)\) are normal and convex, 
\( A \leq B \) and \( A < B \) are defined as follows;

\( A \leq B \iff A \leq B \) is obtained as follows from eqs. (13) and (42).

\[ I_j(y_j) = \pi(y_j \mid u_i^0 \land \ldots \land u_i^0) = \bigvee_{i=1}^{n} c_i(y_i^0)(y_j^0). \]  

(43)

Suppose, however, that \( u_i \) of \( i \) causes \( v_j \) of \( j \), and that \( u_i \) of \( k \) does \( v_j \) of \( j \). Then, if both \( u_i \) and \( u_k \) happen simultaneously, we usually imagine that \( j \) is caused at the larger intensity of \( v_i \) and \( v_j \), as long as there is no canceling nor synergistic effect [9]. Based on this idea with consideration that \( c_i(y_i^0) \) is a fuzzy set on \( V_p \), we give \( I_j \) by the next equation rather than eq. (43).

\[ I_j(y_j) = \bigvee_{i=1}^{n} c_i(y_i^0)(y_j^0). \]  

(44)

This means that we take the maximum of fuzzy sets \( c_i(y_i^0) \), not the maximum of membership values of fuzzy sets \( c_i(y_i^0) \).

We compound the fuzzy intensities of \( j \) based on the above idea, we should use the followings instead of eqs. (12), (13).

\[ \pi(y_j) = \bigvee_{i=1}^{n} \left( \bigvee_{u_i \in U_i} \left( \pi(y_j : u_i) \land \pi(u_i) \right) \right). \]  

(45)

\[ \pi(y_j \mid u_i \land \ldots \land u_p) = \bigvee_{i=1}^{n} \pi(y_j : u_i) \]  

(46)

Though these equations replace eqs. (12) and (13) in this section, eq. (14) still holds.

4.3 Possibilistic Causality Analysis Reasoning with Intensity

We discuss a possibilistic CAR where both \( u_i \) and \( v_j \) are intensities.
4.3.1 Conditional Possibilities of $v_t$

Eq. (15) holds in general. Therefore, so as eq. (18). However, note that calculation of conditional possibilities in eqs. (16), (17) is different from that in the previous section. That is, eq. (13) is used in the previous section, and eq. (46) in this section. Therefore, a set of $v_t$ which gives the largest value of eq. (18) is different from $T$ given by eq. (19). In order to obtain the set of $v_t$, the next proposition should be introduced (Proof is omitted due to lack of space).

[Proposition 4.1]
Possibility distribution on $V_t$ given by the next equation is normal and convex.

$$
\pi(y_t | u^p_t \land \ldots \land u^p_p) = \bigvee_{i=1, p} \pi(y_t | u^p_i),
$$

\begin{equation}
\bigvee_{i=p+1, n} \left\{ \bigvee_{u_{1,i}} \left( \pi(y_t | u_i) \land \pi(u_i) \right) \right\}.
\end{equation}

From eqs. (16), (17) and the above proposition, we understand that there exist vs satisfying $E=F(v)$, and that the set of the vs is an interval whose elements satisfy $\pi(v_t | u^p_t \land \ldots \land u^p_p) \geq E$. If we express the interval in $T$, it is given as follows:

$$
T = \left[ \bigvee_{i=1, n} t^i_{\min}, \bigvee_{i=1, n} t^i_{\max} \right],
$$

$$
[t^i_{\min}, t^i_{\max}] = \left\{ \{v_t | \pi(v_t | u^p_t \land \ldots \land u^p_p) \geq E\} | \bigvee_{u_i; \pi(u_i) \geq E} \bigvee_{i=p+1, n} \left\{ \bigvee_{u_{1,i}} \left( \pi(y_t | u_i) \land \pi(u_i) \right) \right\} \right\}.
$$

4.3.2 Conditional Possibilities of $u_s$

Eq. (21) holds in general, and so as eq. (23). Then, we obtain a set of $u_s$ which gives the largest value of eq. (23). Let $G^{max}$ be the largest value of $G(x)$. It is given as follows:

$$
G^{max} = \bigvee_{u_{1,i} \in U_{1,i}} \left[ \bigwedge_{j=1, q} \left( K_j(y_j) \bigvee \pi(y_j | u^p_j) \right) \right].
$$

$$
K_j(y_j) = \bigvee_{i=1, p} \pi(y_j | u^p_i). \bigvee_{i=p+1, n} \left\{ \bigvee_{u_{1,i}} \left( \pi(y_j | u_i) \land \pi(u_i) \right) \right\}.
$$

The necessary and sufficient conditions of $G(u)=G^{max}$ are clearly given as follows:

$$
\pi(u_s) \geq G^{max}
$$

$$\forall j \in \{1, \ldots, q\}, K_j(y_j) \bigvee \pi(y_j | u_s) \geq G^{max}
$$

Then, the next proposition holds, though the proof is omitted.

[Proposition 4.2]
The set of $u_s$ satisfying eq. (53) is an interval.

From eqs. (50)-(53) and the above proposition, the set of $u_s$ giving the $G^{max}$ is an interval given as follows:

$$
S = \bigcup_{j=0, q} S_j,
$$

$$
S_j = \left\{ \{u_s | \pi(u_s) \geq G^{max}\} \right\}.
$$

5. Numerical Example

A causal model is shown in Fig. 1, where values of $x_t$, $y_t$, and $z_t$ are known, and those of $x_t$, $y_t$, and $z_t$ are not. All of the variables take a value of a totally ordered set $\{a, b, c, d\}$. $\pi(x_t)$ is a prior knowledge for possibilities of intensities of $x_t$. $q$ is an intensity relation between $x_t$ and $y_t$. For example, $q_{11}$ shows $q_{11}(b) = 0.7/a + 1.0/b + 0.7/c + 0.4/d$ when $x_t=b$.

(1) Possibility distribution of $y_t$ given $x_t=b, y_t=c$ and $z_t=b$.

From eq. (46), we obtain the followings:

$$
\pi(y_t | x_t = b) = 0.7/a + 1.0/b + 0.7/c + 0.4/d.
$$

$$
\pi(y_t | x_t = b) = 0.8/a + 1.0/b + 0.4/c + 0.4/d.
$$

$$
\pi(y_t | x_t = b) = 0.6/a + 0.6/b + 0.8/c + 1.0/d.
$$

$E$ and $F(y_t)$ are derived from the above using eq. (16) and (17).

$$
E = \pi(y_t | x_t = b) \land \pi(y_t | x_t = b) \land \pi(y_t | x_t = b) = 0.6.
$$

$$
F(y_t) = \pi(y_t | x_t = b) \land E = 0.6/a + 0.6/b + 0.4/c + 0.4/d.
$$

Therefore, we get the possibility distribution of $y_t$ from eq. (18).

$$
\pi(y_t | x_t = b \land y_t = c \land z_t = b) = 1/a + 1.0/b + 0.4/c + 0.4/d.
$$

Then, we should apply eqs. (48) and (49), we get $T=[a,b]$.

(2) Possibility distribution of $x_t$ given $x_t=b, y_t=c$ and $y_t=b$.

From eq. (46), we obtain the followings:

$$
\pi(y_t | x_t = b \land x_t = a) = 0.7/a + 1.0/b + 0.7/c + 0.4/d.
$$

$$
\pi(y_t | x_t = b \land x_t = b) = 0.7/a + 1.0/b + 0.7/c + 0.4/d.
$$

$$
\pi(y_t | x_t = b \land x_t = c) = 0.7/a + 1.0/b + 0.7/c + 0.4/d.
$$

$$
\pi(y_t | x_t = b \land x_t = d) = 0.7/a + 1.0/b + 0.8/c + 0.6/d.
$$

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the patient’s chronic diseases with their severity. In order to data for the diagnosis are some symptoms with their levels, and diagnosis is uncertain causal relations between severity of disease and its severity. The knowledge we have for the Suppose a medical diagnosis where we have to identify the Therefore, we get

\[ p \in C_1 \times C_2 = \{(b, b) \} \]

In the same way as (2), we get the following:

\[ G(x_2) = \pi(x_1 = c \mid x_1 = b \land x_2) \land (x_2 = b \mid x_2 = b \land x_2) \]

\[ = \pi(x_1 = b) \land \pi(x_2 = b) = 0.1/a + 0.8/b + 0.5/c + 0.1/d. \]

Then, if we apply eq.s (54) and (55), we get \( S = \{b, b\}. \)

(3) Possibility distribution of \( x_1 \) given \( x_2 \): \( x_i = b, y_i = c \) and \( y_i = b \).

In the same way as (2), we get the following:

\[ \pi(x_1 = b \land x_2 = c \land y_2 = b) = 1/a + 1/b + 0.4/c + 0.1/d. \]

\[ S = \{a, b\}. \]

6. CONCLUSIONS

Suppose a medical diagnosis where we have to identify the disease and its severity. The knowledge we have for the diagnosis is uncertain causal relations between severity of possible diseases and levels of possible symptoms. The given data for the diagnosis are some symptoms with their levels, and the patient’s chronic diseases with their severity. In order to solve the diagnosis, we have to deal with both deductive and abductive reasoning as well as uncertainty and intensity (severity and levels). The proposed approach provides us with all of the necessary tools to solve it.

The situation we set in the above diagnosis is not a rare case. We rather believe that it is very common in the real world, and that the proposed reasoning is applicable to many real world problems.

7. REFERENCES


